

Drop Evaporation in a Single-axis Acoustic Levitator

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A 20 kHz single-axis acoustic positioner is used to levitate aqueous-solution drops (volumes \leq 100 micro-liters). Drop evaporation rates are measured under ambient, isothermal conditions for different relative humidities.

Acoustic convection around the levitated sample enhances the mass loss over that due to natural convection and diffusion. A theoretical treatment of the mass flow is developed in analogy to previous studies of the heat transfer from a sphere in an acoustic field.

Predictions of the enhanced mass loss, in the form of Nusselt (Sherwood) numbers, are compared with observed rates of drop shrinking.

The work is part of an ESA study on crystal growth from levitated solution drops.

DROP EVAPORATION IN A SINGLE-AXIS ACOUSTIC LEVITATOR

APPLICATION BACKGROUND: (LABORATORY AND MICRO-G CONDITIONS)

- ESA STUDY ON CRYSTAL GROWTH FROM LEVITATED-SOLUTION DROPS
- LARGER AND BETTER SINGLE CRYSTALS IN CONTAINERLESS PROCESSING

SAMPLE AND ENVIRONMENTAL CONDITIONS:

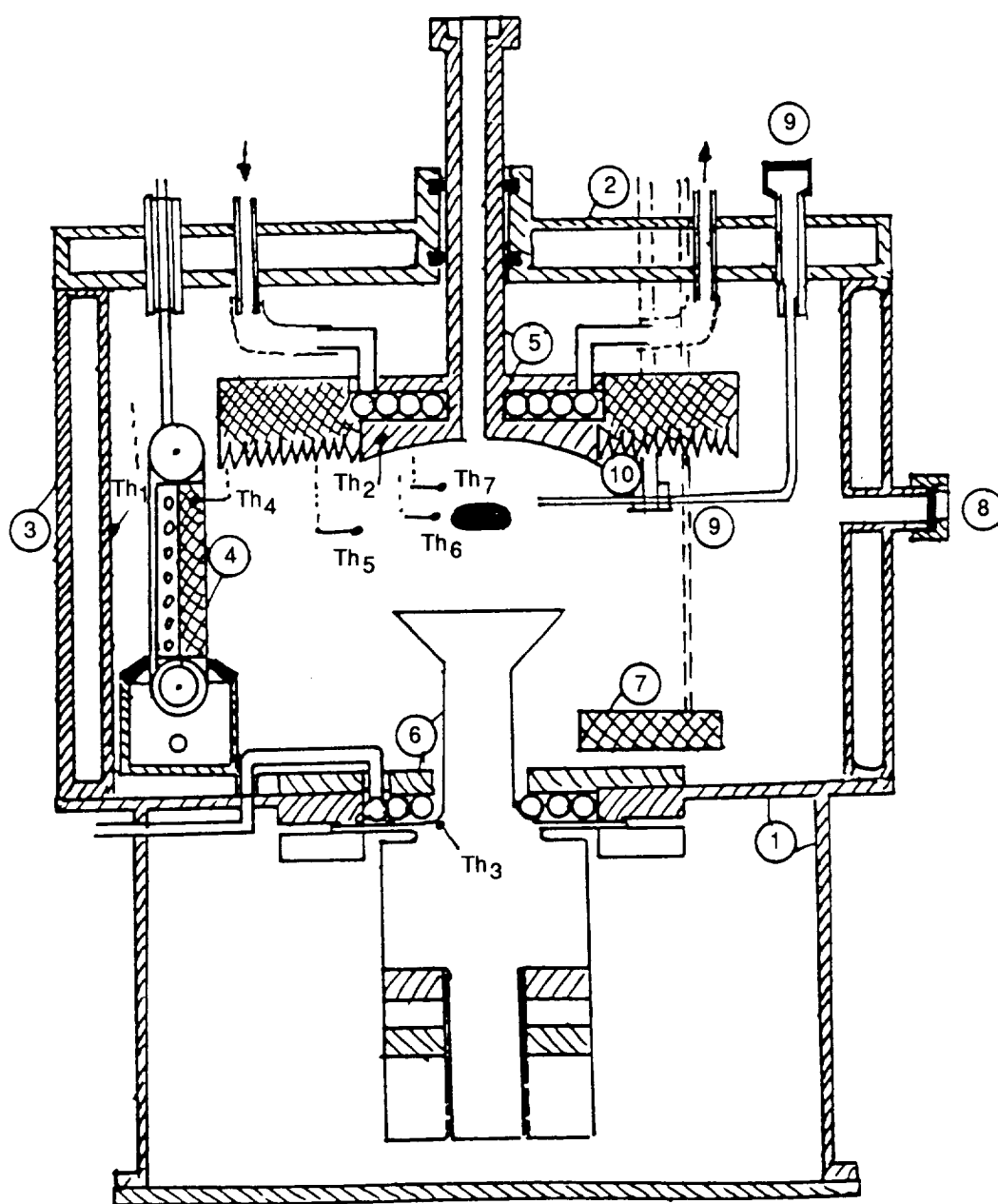
- WATER SOLUTIONS OF INORGANIC AND ORGANIC MATERIALS (PROTEINS), ALSO OTHER SOLVENTS
- DROP SIZE: $10 \text{ ul} < V < 100 \text{ ul}$ ($2.5 \text{ mm} < d < 6 \text{ mm}$)
- ENVIRONMENT: AIR AT AMBIENT PRESSURE (1 atm)
TEMPERATURE: (0 C) $4 \text{ C} < T < 40 \text{ C}$ (70 C)
RELATIVE HUMIDITY: $0 < h_r < 100\%$
- SOUND PRESSURE LEVEL (FOR 1-G): $160 < \text{SPL} < 165 \text{ dB}$

HARDWARE FOR EXPERIMENTS:

- SINGLE-AXIS ACOUSTIC STANDING WAVE LEVITATOR (21 kHz)
- ISOTHERMAL PROCESSING CHAMBER ($T = \pm 0.1 \text{ K}$)
- HUMIDIFIER AND HUMIDITY SENSOR
- CCD CAMERA FOR DROP OBSERVATION AND MONITORING
- SOPHISTICATED OPTICS FOR VISUALIZATION OF STREAMING INSIDE AND OUTSIDE OF THE LEVITATED DROP
- STERILE DROP DEPLOYMENT AND EXTRACTION

Fig. 1 Processing chamber

- | | |
|----------------------------------|-----------------------------------|
| 1 Basic flange and housing | 6 Transducer assembly (cf Fig. 3) |
| 2 Top flange (cf Fig. 5) | 7 Humidity sensor |
| 3 Glass cylinder | 8 Side opening with septum |
| 4 Humidifier (cf Fig. 4) | 9 Sample injector |
| 5 Reflector assembly (cf Fig. 2) | 10 Feeding tube/Manipulator |



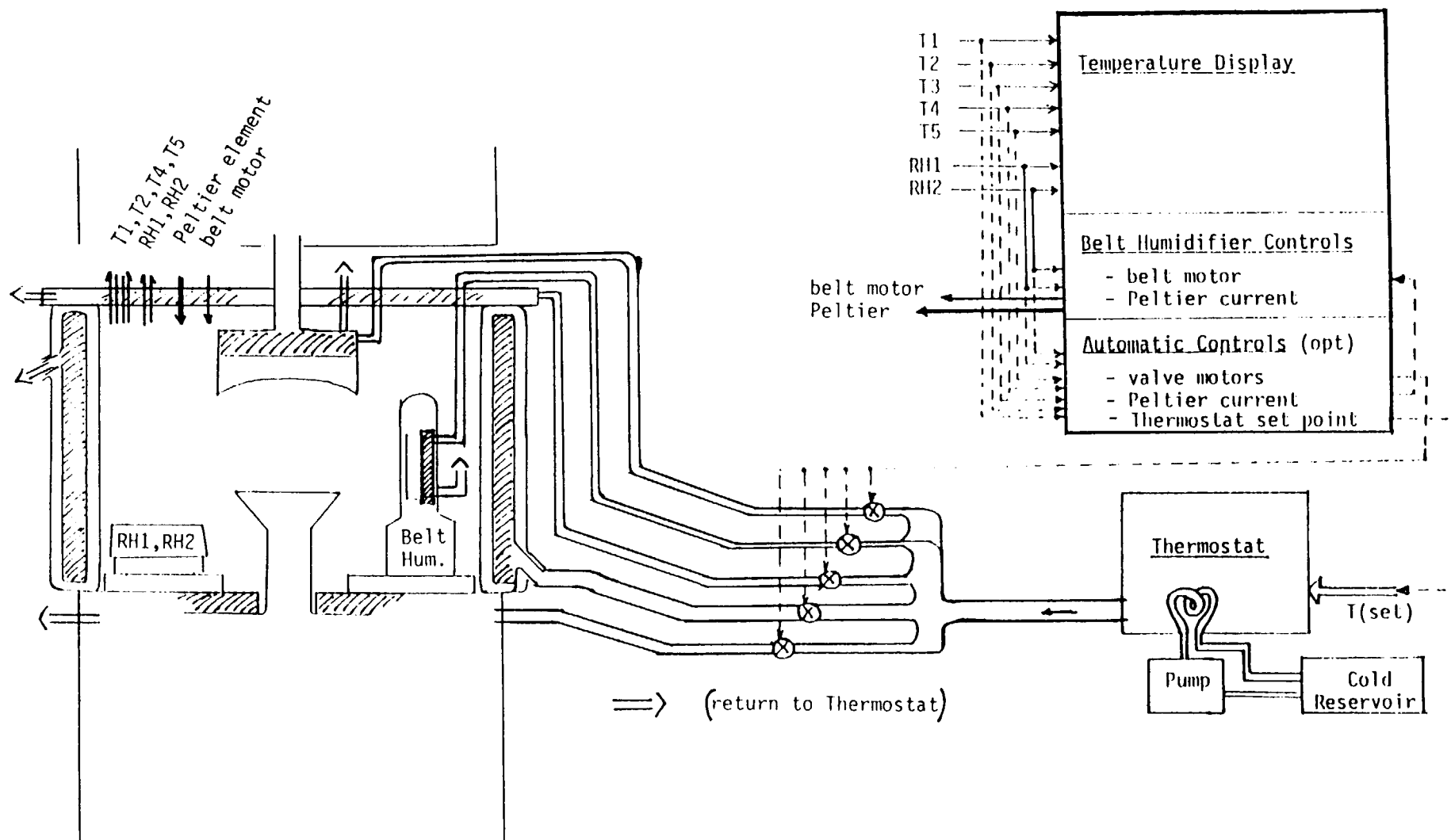


Figure 6. Thermal Control Schematic

DROP EVAPORATION IN A SINGLE-AXIS ACOUSTIC LEVITATOR

EQUATION OF MASS FLOW:

$$\frac{dm}{dt} = -\pi d_s^2 \beta \frac{\rho_v}{R_v T} (1-h_r) \quad (1)$$

$$\beta = Nu \frac{D}{\delta} = Nu \frac{D}{d_s} = Sh \frac{D}{d_s} \quad (2)$$

d_s - DROP DIAMETER

ρ_v - VAPOR PRESSURE OF SOLVENT

R_v - GAS CONSTANT

T - TEMPERATURE

h_r - RELATIVE HUMIDITY

D - GAS DIFFUSION CONSTANT

δ - BOUNDARY LAYER THICKNESS

Nu - NUSSELT NUMBER (TOTAL HEAT FLOW)/(CONDUCTIVE HEAT FLOW)

Sh - SHERWOOD NUMBER (TOTAL MASS FLOW)/(DIFFUSIVE MASS FLOW)

BASIC THEORY OF DROP EVAPORATION (IN ANALOGY TO HEAT FLOW)

A FREE DROP IN AN ISOTHERMAL ENVIRONMENT AT A RELATIVE HUMIDITY, $h_r < 1$, HAS SATURATED HUMIDITY ($h_r = 1$) INSIDE THE BOUNDARY LAYER. DENSITY DIFFERENCES BETWEEN THE BOUNDARY LAYER AND THE BACKGROUND RESULT IN NATURAL CONVECTION ($Nu = Sh > 2$) WHICH IS FURTHER ENHANCED UNDER ACOUSTIC LEVITATION CONDITIONS BY STREAMING. (FIG. 2)

EQUATION (1) LEADS TO A SIMPLE NORMALIZED EQUATION FOR THE DROP DIAMETER AS A FUNCTION OF TIME

$$\left[\frac{d_s}{d_{s,0}} \right]^2 = \frac{d_s^2}{d_{s,0}^2} = 1 - 0.37 \bar{t} \quad (3)$$

WITH $\bar{t} = t / t_{0.5}$, AND

$$t_{0.5} = \frac{9.25 \times 10^{-2} d_{s,0}^2 R_v T \rho_s}{D p_v (1-h_r) Nu} \quad (4)$$

WHERE $t_{0.5}$ IS THE TIME REQUIRED FOR A DROP TO SHRINK TO 50% OF ITS INITIAL VOLUME.

THE REFERENCE TIME, $t_{0.5}$, (EQUATION (4)) CONTAINS KNOWN TEMPERATURE DEPENDENT PROPERTIES OF THE SOLVENT. THE ONLY UNKNOWN IS THE NUSSELT (OR SHERWOOD) NUMBER.

$$t_{0.5} [h_r] \approx \frac{24}{Nu} \frac{d_{s,0}^2}{1-h_r} \left(\frac{T}{T_0} \right)^{-0.94} e^{-19.7(1-\frac{T_0}{T})}$$

for H₂O drops in air

SOLUTION DROPS

WHEN THE DROP CONTAINS A "SALT" SOLUTION, WITH CONCENTRATION c_a , AND THE PROCESSING CHAMBER CONTAINS A LIQUID RESERVOIR WITH THE SAME SOLUTION BUT A LARGER OR SMALLER CONCENTRATION, c_∞ , THAN EQUATION (1) HAS TO BE MODIFIED BECAUSE THE VAPOR PRESSURE (HUMIDITY) IN THE DROP BOUNDARY LAYER, c_a , AND IN THE ENVIRONMENT ARE REDUCED ACCORDING TO RAOULT'S LAW. (FIG.3)

THE NORMALIZED DROP DIAMETER, $\bar{d}_s = d_s / d_{s,0}$, AS A FUNCTION OF THE NORMALIZED TIME IS GIVEN BY

$$2 \frac{\bar{d}_s d(\bar{d}_s)}{1 - c_a \bar{d}_s^3} = -0.37 \frac{1 - h_r c_\infty}{1 - h_{r,0}} d\bar{t} \quad (5)$$

WITH $h_r(c_\infty)$ BEING THE RELATIVE HUMIDITY OF THE "SALTY" BACKGROUND. FIG. 5 SHOWS THE SHRINKING CURVES FOR DIFFERENT CONCENTRATION RATIOS, c_a / c_∞ , AND INDICATES LIMITED DROP SHRINKING FOR $c_a / c_\infty < 1$ AND LIMITED DROP GROWTH FOR $c_a / c_\infty > 1$ RESULTING FROM LIMITED CONCENTRATION CHANGES INSIDE THE DROP.

$$\frac{t_{0.5}}{[h_r]} \approx \frac{24}{Nu} \frac{d_{s,0}^2}{1 - h_r} \left(\frac{T}{T_0} \right)^{-0.94} e^{-19.7(1 - \frac{T_0}{T})}$$

$1 - h_r \approx c_{s,\infty}$ cf fig. 3

NUSSELT NUMBER MODEL

BECAUSE OF THE ANALOGY BETWEEN HEAT FLOW AND MASS FLOW OF A LEVITATED SAMPLE, WE CAN REFER TO THE EXTENSIVE THERMAL INVESTIGATIONS BY C.P. LEE AND T. WANG [1] AND BY E. LEUNG [2]. THESE AUTHORS FOUND AN EXPERIMENTALLY PROVEN CONNECTION BETWEEN THE NUSSELT NUMBER, Nu , THE GRASHOF NUMBER, Gr , (THE RATIO OF BUOYANCY AND VISCOSITY EFFECTS) AND THE EFFECTIVE REYNOLDS NUMBER, Re , OF A LEVITATED SAMPLE RESULTING FROM FORCED CONVECTION IN THE ENVIRONMENT AROUND THE SAMPLE. (FIG. 5)

FOR RELATIVELY LARGE SPL AND RESULTING REYNOLDS NUMBERS, E. LEUNG FOUND AS A GOOD APPROXIMATION FOR THE HEAT FLOW (FIG. 6),
FOR $10 < Re < 50$

$$Nu = 2 + e^A \cdot Gr^B$$

WITH $A = -0.72 + 0.46 \ln(1 + Re)$
AND $B = 0.25 - 0.015 \ln(1 + Re)$.

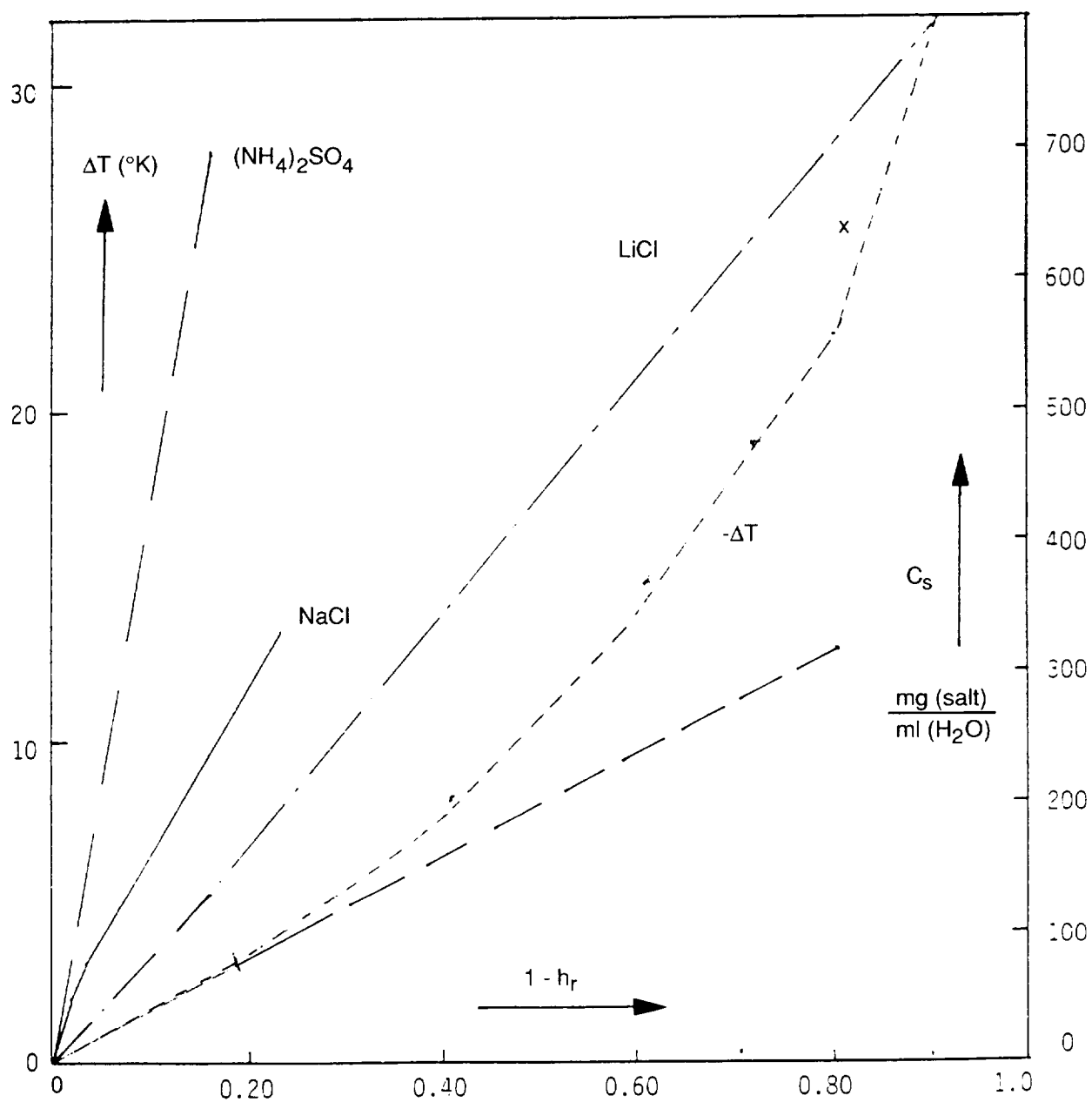


Figure 3 Required undercooling and salinity of the fluid reservoir for subsaturated humidity in the processing chamber at 20°C .

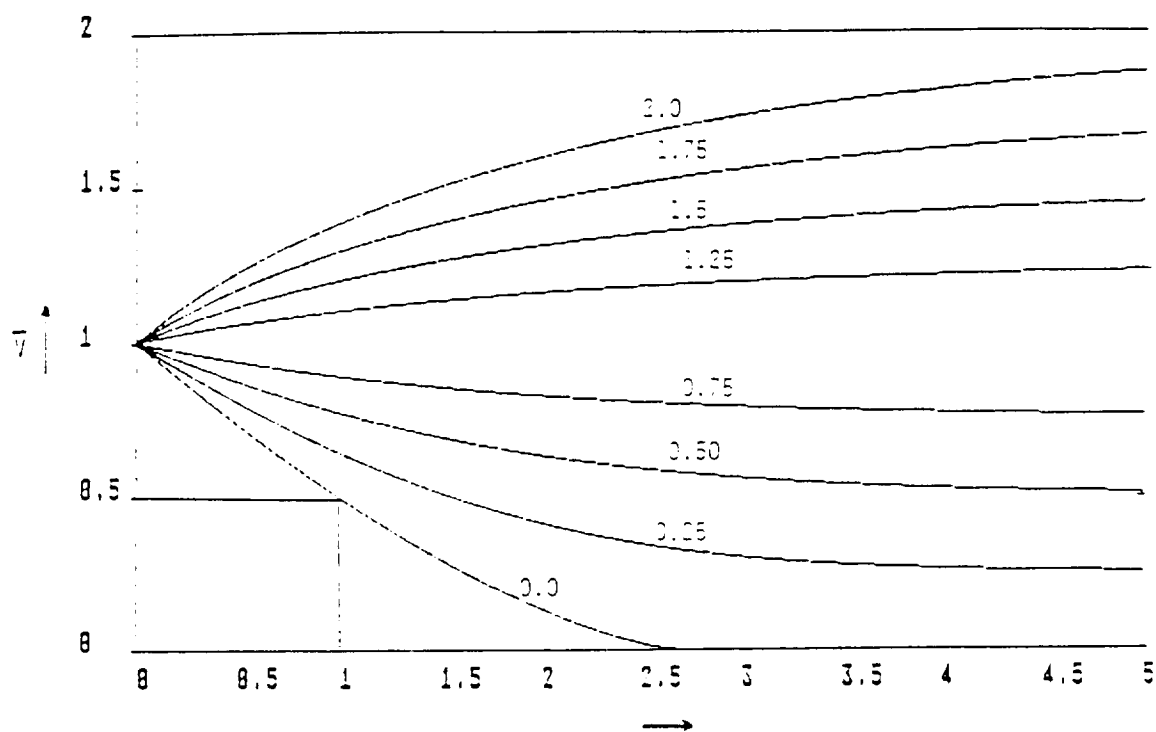


Figure 4. Normalized drop volume, $\bar{V} = V_s/V_{5.03}$ versus normalized time, $\bar{t} = t/t_{1/2}$, for different values of the ratio c_a/c_b . The C_a is the salt concentration in the drop and c_b is its concentration in the fluid reservoir.

GRASHOFF NUMBER MODEL

THE GRASHOFF NUMBER FOR A SOLUTION DROP WITH SATURATED VAPOR IN ITS BOUNDARY LAYER LEVITATED IN AN ISOTHERMAL ENVIRONMENT OF A RELATIVE HUMIDITY OF $h_r < 1$, CAN BE EXPRESSED AS

$$Gr = d_s^2 \frac{\Delta \rho}{\rho_\infty} \frac{1-h_r}{\nu^2} g$$

d_s - SAMPLE DIAMETER
 ρ_∞ , $\Delta \rho$ - AMBIENT GAS DENSITY,
EXCESS DENSITY IN BOUNDARY LAYER
 h_r - RELATIVE HUMIDITY OF GAS
 ν - KINEMATIC VISCOSITY
 g - GRAV. ACCELERATION ($g = 9.81 \text{ m/s}^2$)

FOR WATER DROPS IN AIR THE GRASHOFF NUMBER CAN BE APPROXIMATED BY

$$Gr = 208 d_s^3 (1 - h_r) e^{\alpha(T) \cdot T}$$

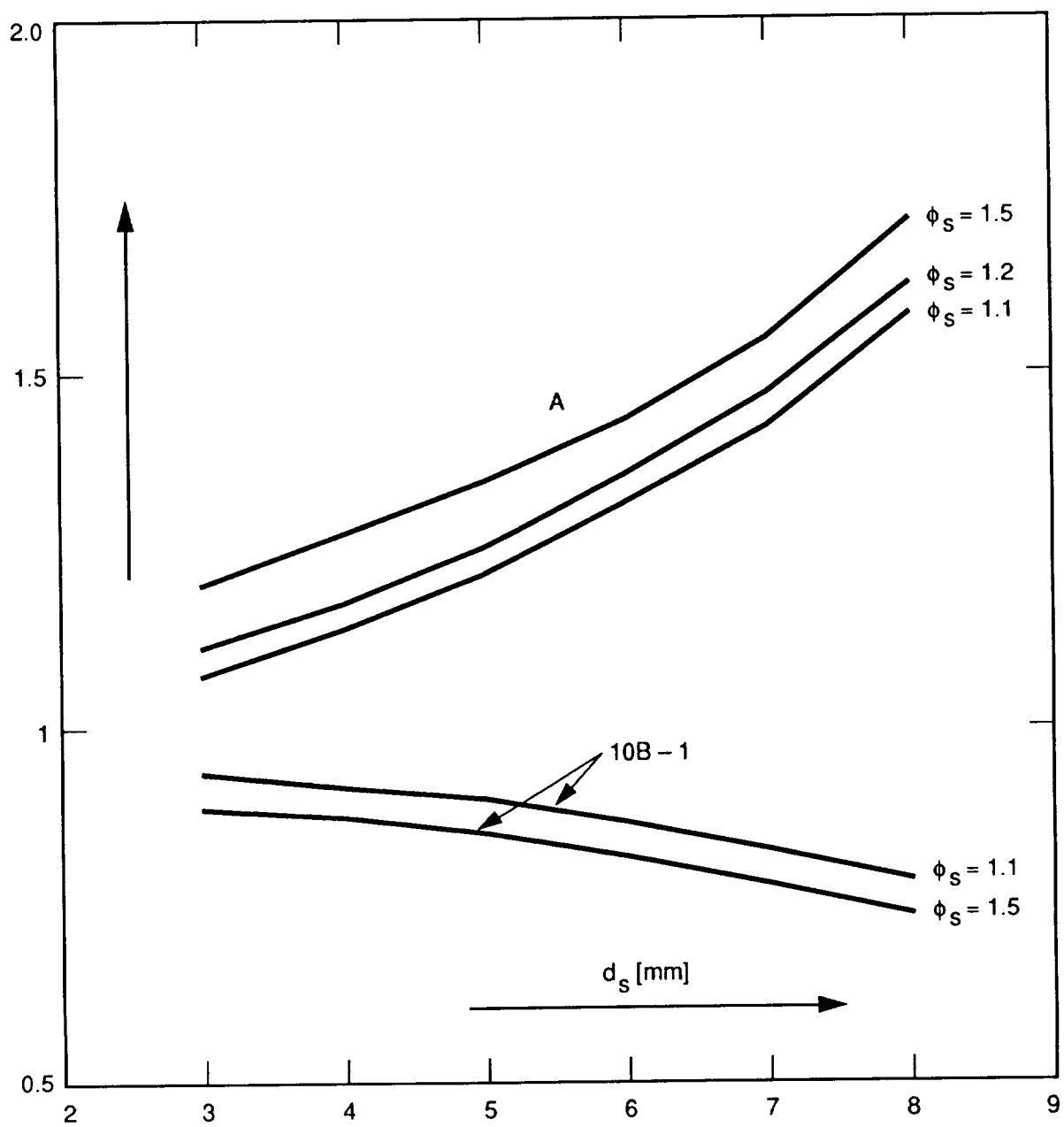
WITH $\alpha = 0.058 (1 - 0.0033T)$ AND d_s MEASURED IN cm.

DISCUSSION OF EXPERIMENTAL RESULTS

WHEN INSERTING TYPICAL REYNOLDS NUMBERS, Re , FOR WATER DROPS WITH DIAMETERS BETWEEN 2 AND 6mm, LEVITATED IN AMBIENT AIR AT 20 KHZ (TABLE 1), WE FIND NUSSELT NUMBERS BETWEEN 5 AND 10 DEPENDING ON DROP DIAMETER d , RELATIVE HUMIDITY, h_r , TEMPERATURE, T , AND SPL (OR LEVITATION SAFETY FACTOR, ϕ_s). FOR CONSTANT LEVITATION SAFETY FACTOR, ϕ_s , Nu INCREASES LINEARLY WITH T AND d .

FIGURE 7 SHOWS A TYPICAL DROP SHRINKING CURVE MEASURED AT 20 C AND A RELATIVE HUMIDITY OF ABOUT 80%. IN THE DISCRETE RANGE BETWEEN 2 AND 3 mm, THE CALCULATED NUSSELT NUMBER IS $Nu = ______$ WHICH DIFFERS BY A FACTOR OF 1.3 FROM THE MEASURED VALUE; IT MAY RESULT FROM UNCERTAINTIES IN THE HUMIDITY MEASUREMENT.

WHEN THE DROP DIAMETER, TEMPERATURE, SPL, AND RELATIVE HUMIDITY ARE ONLY SLIGHTLY VARIED DURING A MEASUREMENT THE NUSSELT NUMBER CAN BE ASSUMED CONSTANT. IN THIS CASE IT IS POSSIBLE TO PREDICT THE RELATIVE SHRINKING TIME, $t_{0.5}$, (EQUATION 4), FOR A GIVEN ACCURACY OF THE MEASURED HUMIDITY, h_r , AND TEMPERATURE, T .



$$Nu \approx 2 + e^A (1 + Gr)^B$$

$$Gr \approx 208 d_s^3 (1 - hr) e^{\alpha(T) T} \quad d \text{ [cm]}$$

$$d \approx 0.058 (1 - 0.0033T) \quad T \text{ [}^\circ\text{C]}$$

FIG. 6

diameter (mm)	2	3	4	5	6
Re	46	49.6	56	65.4	78.4
Re^A	2.86	2.96	3.13	3.35	3.63
B	0.192	0.191	0.189	0.187	0.184
Gr (20 C)	4.9	16.5	39	76	132
Nu ($h_r = 0$)	5.9	7.0	8.3	9.5	10.9
Nu ($h_r = 0.8$)	4.8	5.7	6.6	7.5	8.5

